

# A hybrid Measurement Systems Analysis and Uncertainty of Measurement Approach for Industrial Measurement in the Light Controlled Factory

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**Abstract** *The uncertainty of measurements must be quantified and considered in order to prove conformance with specifications and make other meaningful comparisons based on measurements. While there is a consistent methodology for the evaluation and expression of uncertainty within the metrology community industry frequently uses the alternative Measurement Systems Analysis methodology. This paper sets out to clarify the differences between uncertainty evaluation and MSA and presents a novel hybrid methodology for industrial measurement which enables a correct evaluation of measurement uncertainty while utilising the practical tools of MSA. In particular the use of Gage R&R ANOVA and Attribute Gage studies within a wider uncertainty evaluation framework is described. This enables in-line measurement data to be used to establish repeatability and reproducibility, without time consuming repeatability studies being carried out, while maintaining a complete consideration of all sources of uncertainty and therefore enabling conformance to be proven with a stated level of confidence. Such a rigorous approach to product verification will become increasingly important in the era of the Light Controlled Factory with metrology acting as the driving force to achieve the right first time and highly automated manufacture of high value large scale products such as aircraft, spacecraft and renewable power generation structures.*

## 1. Introduction

No measurement is ever completely certain; the result is always an estimate of the true value with some degree of uncertainty. The result of a measurement should always be accompanied by a quantitative indication of its uncertainty allowing it to

be compared with references or specifications and the methodology for uncertainty evaluation is well established by international standards [1]. If the uncertainty of a measurement is high then it becomes likely that it will erroneously show non-conformance for a conforming part or visa-versa. The role of uncertainty in proving conformance or non-conformance of parts with specifications is central to the rapidly developing Geometric Product Specification (GPS) standards [2] and will become increasingly important in the era of the Light Controlled Factory (LCF) where metrology will enable the right first time and highly automated manufacture of products such as aerospace and renewable power generation structures [3, 4].

*Six Sigma* is a popular process improvement methodology developed by *Motorola* in the 1980's [5] within which each business function is defined as a process with **measurable** Critical to Quality (CTQ) characteristics. Variances in measurements of the CTQ's are compared with the customer's CTQ limits to determine the expected rate of quality defects. Such a measurement focused methodology clearly requires consideration of the uncertainty of measurements and the recommended method is *Measurement Systems Analysis* (MSA). MSA includes tools such as Gage Repeatability and Reproducibility studies (Gage R&R) and, although not a complete uncertainty evaluation, gage R&R is widely used to evaluate measurements in industry.

This paper sets out to clarify the differences between uncertainty evaluation and MSA and presents a hybrid methodology which enables a correct evaluation of measurement uncertainty while utilizing the practical tools of MSA. Previous work considering such a hybrid approach has been focused on inter-laboratory comparisons [6, 7] while this paper presents a practical method for industrial use.

## **2. Uncertainty Evaluation and Measurement Systems Analysis**

A number of standards are referenced extensively within this work; The Guide to the Expression of Uncertainty in Measurement (GUM) [1] defines uncertainty evaluation within the metrology community; ISO 14253-2 [8] provides a practical method for applying the GUM to industrial measurements; ISO 5725 [9] defines accuracy (trueness and precision) within MSA; and the Automotive Industry Action Group (AIAG) MSA manual [10] is the primary reference for MSA in industry.

### **2.1. Differences in Vocabulary**

Definitions for key terms in uncertainty evaluation and MSA are often somewhat conflicting. Historically *accuracy* has been used to describe how close the mean of many measurements is to a known reference value. Its use is now changed. In the GUM it is replaced by the term *Uncertainty of Measurement* which includes all sources of uncertainty. In ISO 5725 however the historical meaning of accuracy is

replaced by *Trueness* while *Accuracy* is now used to refer to the combination of *Precision* and *Trueness*. Table 1 summarizes the important differences.

Table 1: Comparison of Terms in Uncertainty Evaluation (GUM) and MSA (ISO 5725)

Word	GUM	ISO 5725
Accuracy	Uncertainty is now the quantitative term	Combination of Precision and Trueness
Precision	Random Uncertainty	Combination of Repeatability and Reproducibility
Repeatability	Variation/Precision where results are obtained under the same conditions	
Reproducibility	Variation in measurements of the same part under changed conditions. The conditions which were changed should be stated; principle of measurement; method of measurement; observer; measuring instrument; reference standard; location; conditions of use; or time.	Precision where results are obtained with the same method on identical test items in different laboratories with different operators using different equipment.
Bias	Systematic Uncertainty (Term considered misleading, to be avoided)	Replaced by Trueness
Trueness		Difference between the mean of many measurements and the reference value
Uncertainty	Used for both the general concept of doubt in the validity of a measurement result and for all quantitative measures of this.	Accuracy is the equivalent term.

## 2.2. Fundamental Differences

When quantifying the uncertainty of a measurement the aim is to establish a range of values within which we have confidence that the true value lies. Therefore all the factors affecting the measurement result must be considered and their effect on the measurement result quantified. Typical factors affecting measurements include:

- Uncertainty of the reference used to calibrate the instrument; with an unbroken chain of calibrations providing traceability to primary standards.
- Random variation in use (repeatability)
- Differences in results from different operators and conditions (reproducibility)
- Environmental uncertainty (uncertainty in temperature used for compensation)
- Uncertainties in alignments and setup parameters
- Rounding errors

Components of uncertainty for these factors are classified as Type A obtained by statistical analysis of a series of observations or Type B obtained by other means. The components may also be classified as either *random* (affecting *precision*) or *systematic* (affecting *trueness*). Regardless of classification all components are modelled by probability distributions quantified by their variance and combined to give a total uncertainty. The total uncertainty is multiplied by a coverage factor to give bounds to the possible range of values within which the true value may lie, at a given confidence level. This is known as the *expanded uncertainty*.

This uncertainty evaluation approach is sometimes called a *bottom-up* approach since it considers each source of uncertainty individually. It depends on a realistic mathematical model for the way in which these sources of uncertainty will

contribute to the combined uncertainty. The iterative Procedure for Uncertainty Management (PUMA) [8] is a practical approach to the GUM suitable for industrial measurements. PUMA involves initially calculating combined uncertainty based on quick 'worst case' estimates where this is convenient, establishing which sources are significant and attempting to reduce these until a satisfactory value is obtained.

In MSA instruments should be calibrated but it is assumed that the calibration uncertainty is negligible. The random variation in the measurement process is then compared with the part-to-part variation using Gage R&R studies (detailed below). There are recommendations that the repeatability and reproducibility (total Gage R&R) should be some fraction of the process variation or tolerance. There is no explicit consideration of other sources of uncertainty. This approach is sometimes called a *top-down* approach since it considers the output of the complete process without consideration to the individual input quantities.

In both uncertainty evaluation and MSA all influences on the measurement result must be considered. In uncertainty evaluation they must be explicitly included in the model and in MSA reproducibility conditions must properly represent them. Thermal variation often dominates the uncertainty of industrial measurements [11] and there is a risk that in MSA it will not be fully represented by reproducibility conditions. MSA does not normally consider the uncertainty of the reference standard, the uncertainty in the calculated trueness or bias value, although it is included in the draft document ISO/TS 21748. Where MSA considers the uncertainty in the reference standard, and the reproducibility conditions are fully representative, it may be considered to give a full and traceable evaluation of the uncertainty of measurement.

Uncertainty evaluation requires carefully designed uncertainty evaluations planned and carried out by highly skilled engineers. The resulting uncertainty can be applied to decision rules for proving conformance or non-conformance with specifications to accept or reject parts based on a statistical confidence level. MSA provides standard tools for process improvement which can be applied to any process by production engineers, provided the right analysis software is available. It does not fully consider all sources of uncertainty and does not provide acceptance or rejection decisions which are based on statistical confidence.

### **3. Practical Uses of MSA in Industry**

A number of different gage studies are used within MSA to establish the variation due to instruments. For a variable gage (one which gives a measurement value) a Type 1 Gage Study is normally carried out first to determine the capability of a measurement process by evaluating its bias and repeatability using measurements

of a single part and comparing these with the component's tolerance for which conformance must be determined.

A Gage R&R study is then carried out to identify any deficiencies in the measurement system. This is a multi-factor experiment in which the variance components can be determined for part variation, instrument repeatability, operator reproducibility and in some cases part-operator interaction. It evaluates how much of the observed process variation is due to measurement system variation and how much is due to actual variation in the parts being measured.

There are a number of different types of Gage R&R studies with different experimental designs. The simplest of these is the Crossed study in which each part is measured the same number of times by all operators; a balanced design. There are also nested studies primarily used for destructive testing and expanded studies where it is possible to include more factors than operator and part, fixed factors, a mixture of crossed and nested factors or an unbalanced design.

Attribute Gage studies are used to determine the bias and repeatability of measurements which give a pass or fail result as opposed to a variable output. For example Go/No-Go gages and visual inspection for defects. Attribute gage studies are used to evaluate the bias and repeatability of a gages which output a binary attribute variable, such as pass or fail. Examples of attribute gages are gap gages, plug gages and visual inspection processes.

### **3.1. Type 1 Gage Studies**

A Type 1 gage study involves a single operator measuring a single part a number of times. A minimum of 10 and preferably at least 25 replicates of the measurement are normally recommended [12]. Simple statistics are then calculated from the measurement results such as the mean and the standard deviation for the sample. This is essentially a repeatability study as would often be carried out as part of an uncertainty evaluation.

The part being measured should also be calibrated against a reference instrument so that the difference between this reference value and the mean of the measurements gives an indication of the measurement system bias. This provides a degree of traceability but since the uncertainty of the reference standard is not considered, and other sources of uncertainty such as temperature are also not considered, this may not be considered a traceable uncertainty evaluation. A T-test may be carried out to determine whether there is a statistically significant difference between the reference value and the mean of  $n$  measurements.

### 3.2. Gage R&R Studies

Variation in measurements (precision) is affected by many factors such as; the operator; equipment used; calibration; environment; and the time elapsed between measurements. As more of these factors vary so measurement variation increases. There are two extreme conditions of precision; repeatability is the minimum condition where the above factors are constant; and reproducibility is the maximum condition where all of these factors vary. Often all of the factors effecting reproducibility are not varied since some of these will be constant for the process under consideration.

In a typical Crossed Gage R&R study a number of parts, typically 10, are each measured a few times by each of a few different operators. By applying Analysis of Variance (ANOVA) it is then possible to determine the individual variance components due to the part variation, the repeatability of measurements and the reproducibility between different operators. The Grand Mean is first calculated which is simply the mean for all measurement values. The sums of the squared differences (SS) from this grand mean are then calculated with respect to the part ( $SS_{Part}$ ), operator ( $SS_{Op}$ ), repeatability ( $SS_{Rep}$ ) and total variation ( $SS_{Total}$ ) using

$$SS_{Part} = n_{Op} \cdot n_{Rep} \sum (\bar{x}_{i...} - \bar{x})^2 \quad (1) \qquad SS_{Op} = n_{Part} \cdot n_{Rep} \sum (\bar{x}_{j...} - \bar{x})^2 \quad (2)$$

$$SS_{Rep} = \sum \sum \sum (x_{ijk} - \bar{x}_{ij})^2 \quad (3) \qquad SS_{Tot} = \sum (x_{ijk...} - \bar{x})^2 \quad (4)$$

where  $n_{Op}$  is the number of operators,  $n_{Rep}$  is the number of replicate measurements of each part by each operator,  $n_{Part}$  is the number of parts,  $\bar{x}$  is the grand mean,  $\bar{x}_i$  is the mean for each part,  $\bar{x}_j$  is the mean for each operator,  $x_{ijk}$  is each observation and  $\bar{x}_{ij}$  is the mean for each factor level.

All of the Gage R&R calculations, including finding these sums of the squared differences, can be easily calculated in a spreadsheet. An example spreadsheet with a full explanation is available online [13].

The sum of the squared differences for part by operator interaction is the residual variation given by

$$SS_{Part*Op} = SS_{Tot} - SS_{Part} - SS_{Op} - SS_{Rep} \quad (5)$$

The numbers of different parts ( $n_{Part}$ ), operators ( $n_{Op}$ ) and replicate measurements ( $n_{Rep}$ ) are used to calculate the degrees of freedom ( $DF$ ) for each factor using

$$DF_{Part} = n_{Part} - 1 \quad (6) \qquad DF_{Op} = n_{Op} - 1 \quad (7)$$

$$DF_{Rep} = n_{Part} \cdot n_{Op} \cdot (n_{Rep} - 1) \quad (8) \qquad DF_{Tot} = n_{Part} \cdot n_{Op} \cdot n_{Rep} - 1 \quad (9)$$

$$DF_{Part*Op} = (n_{Part} - 1)(n_{Op} - 1) \quad (10)$$

The mean squared difference for each factor is calculated by dividing the sum of the squared differences for this factor by the corresponding degrees of freedom. The significance of the part-by-operator interaction on variation should then be determined by first calculating the F-statistic for this factor using

$$F_{Part*Op} = \frac{MS_{Part*Op}}{MS_{Rep}} \quad (11)$$

The probability of  $F_{Part*Op}$  being significant is found from an F-distribution. If the interaction is significant then the above values of the mean squared differences are used to calculate components of variance. If the interaction is not significant then the same values are used for  $MS_{Part}$  and  $MS_{Op}$  but  $MS_{Part*Op}$  is ignored and  $MS_{Rep}$  is now the residual variation and therefore  $SS_{Rep}$  is calculated as

$$SS_{Rep} = SS_{Tot} - SS_{Part} - SS_{Op} \quad (12)$$

Variance components for each factor can now be calculated; the component for part-to-part variation ( $\sigma_{Part}^2$ ) is calculated using equation (13) when the part by operator interaction is significant and equation (14) when it is not significant.

$$\sigma_{Part}^2 = \frac{MS_{Part} - MS_{Part*Op}}{n_{Op} \cdot n_{Rep}} \quad (13) \quad \sigma_{Part}^2 = \frac{MS_{Part} - MS_{Rep}}{n_{Op} \cdot n_{Rep}} \quad (14)$$

The variance component for operator variation ( $\sigma_{Op}^2$ ) is calculated using equation (15) when the part by operator interaction is significant and equation (16) when it is not significant.

$$\sigma_{Op}^2 = \frac{MS_{Op} - MS_{Part*Op}}{n_{Part} \cdot n_{Rep}} \quad (15) \quad \sigma_{Op}^2 = \frac{MS_{Op} - MS_{Rep}}{n_{Part} \cdot n_{Rep}} \quad (16)$$

The variance component for repeatability ( $\sigma_{Rep}^2$ ) is calculated using equation (17).

$$\sigma_{Rep}^2 = MS_{Rep} \quad (17)$$

The variance component for part by operator interaction ( $\sigma_{Part*Op}^2$ ) is only calculated when this factor is significant and is given by

$$\sigma_{Part*Op}^2 = \frac{MS_{Part*Op} - MS_{Rep}}{n_{Rep}} \quad (18)$$

When part by operator interaction is not significant the variance component for reproducibility ( $\sigma_{Reprod}^2$ ) is equal to  $\sigma_{Op}^2$ . When interaction is significant it is given by

$$\sigma_{Reprod}^2 = \sigma_{Op}^2 + \sigma_{Part*Op}^2 \quad (19)$$

The total Gage R&R ( $\sigma_{GRR}^2$ ) is the sum of repeatability and reproducibility.

$$\sigma_{GRR}^2 = \sigma_{Rep}^2 + \sigma_{Reprod}^2 \quad (20)$$

The total process variation ( $\sigma_{Tot}^2$ ) is the sum of total Gage R&R and part-to-part variation.

$$\sigma_{Tot}^2 = \sigma_{GRR}^2 + \sigma_{Part}^2 \quad (21)$$

The standard deviations for each factor are simply the square root of the corresponding variance component.

### 3.3. Attribute Gage Studies

Attribute gage studies are used for gages such as gap gages, plug gages and visual inspection processes which output a binary attribute variable, such as pass or fail. This study can evaluate the bias and repeatability of an attribute gaging process and determine its capability to determine whether a part conforms to specifications.

It is very important to select parts very carefully for an attribute gage study; the parts should be at close to equidistant intervals and should have been accurately measured to determine their reference values. For an AIAG standard study each part should be measured at least 20 times. The probability of each part passing or failing the gaging process is calculated from these results and recorded in a table alongside the calibrated reference value. A regression analysis can then be used to fit the normal distribution to this data and therefore calculate the mean and standard deviation. The mean is used to test for significant bias and the standard deviation is the total Gage R&R variation including in an uncertainty budget in the normal way. This process can be better understood from an example as given in section 4.2 below.

### 4. Hybrid Uncertainty Evaluation using MSA Tools

The hybrid approach to uncertainty evaluation described in this paper involves the following steps:

1. Carry out a bottom up evaluation of sources of uncertainty.
2. Design a Gage R&R study to include as many sources of uncertainty in the reproducibility conditions as can be practically representative of the process.
3. Establish reference values for the parts used in the Gage R&R study using a calibrated reference and include the uncertainty in the reference.



4. Determine the mean bias of the measurement process and use a T-test to determine whether it is significant or a result of the random variation (total Gage R&R). If bias is significant then it is added to the expanded uncertainty.
5. Calculate the combined and expanded uncertainty including the total Gage R&R standard deviation, the uncertainty of the calibration in step 3, any uncertainties not included in the reproducibility conditions and any significant bias.
6. Set conformance limits by adding the expanded uncertainty to the lower specification limit and subtracting the expanded uncertainty from the upper specification limit.
7. Determine the percentage of conforming parts which the process will falsely reject.
8. If the combined uncertainty results in an unacceptable rate of conforming part rejection then consider which sources of uncertainty have a significant effect on the combined uncertainty and either obtain improved estimates for them or improve the process to minimize them.

This section gives two examples of the practical use of a Hybrid MSA and Uncertainty of Measurement approach to prove conformance of parts in industrial processes. The first example considers parts which are being produced on a 5-axis machine tool and for which 100% inspection is then carried out using on-machine probing. The second example considers a visual inspection process for defects in fabric preforms.

#### **4.1. Example 1: Variable Gaging using On-Machine Probing**

A process is considered in which parts are produced on a 5-axis machine tool with 100% inspection carried out using on-machine probing. Uncertainty evaluation must minimize machine down time. A critical dimension, with nominal length 100 mm and a tolerance of  $\pm 40 \mu\text{m}$ , is observed. A Gage R&R Crossed Study is carried out in which three machines are selected for the study, each machine is set to repeat its standard measurement cycle 3 times at regular intervals throughout the day. Each machine measures 10 parts which are subsequently measured on a high accuracy CMM. Measurements are carried out across the day to include in the reproducibility conditions the effects of machine warm up and temperature variation. It is accepted that seasonal variation in temperature and machine drift between calibrations are not included in the reproducibility conditions of the initial study but it is planned to add additional data later in the year to include this.

The Gage R&R study finds a total Gage R&R standard deviation for the process of  $5 \mu\text{m}$ . The mean bias is  $1 \mu\text{m}$  which is significant. Since annual temperature variation and machine drift between calibrations are not included in the reproducibility conditions these are included as individual terms in the initial

uncertainty budget. The machine drift is estimated based on calibration records for the machines. The seasonal temperature variation is based on worst case estimates. The combined uncertainty is then calculated using the values in Table 2.

Table 2: Initial Uncertainty Budget Variable Gaging using On-Machine Probing

Uncertainty Source	Value	Distribution	Standard Uncertainty
Total Gage R&R	5 $\mu\text{m}$	Normal	5 $\mu\text{m}$
Reference CMM	1 $\mu\text{m}$	Normal	1 $\mu\text{m}$
Thermal Variation	15 $\mu\text{m}$	Normal	15 $\mu\text{m}$
Machine Drift	2 $\mu\text{m}$	Normal	2 $\mu\text{m}$
Combined Standard Uncertainty			16 $\mu\text{m}$
Bias			1 $\mu\text{m}$
Expanded Uncertainty (inc. Bias, k=2)			33 $\mu\text{m}$

Based on the initial estimate for the expanded uncertainty the conformance limits are set at +/- 7  $\mu\text{m}$ , a large reduction from the specification limits of +/- 40  $\mu\text{m}$ .

The machine tool variation has a standard deviation of 4  $\mu\text{m}$  giving a process capability of just 0.59 with respect to the conformance limits. More informatively the conformance limits are within 1.77 standard deviations resulting in 7.7% of conforming parts being rejected which is considered unacceptable! A target of 0.1% part rejection is set which is calculated to require an expanded uncertainty of 27  $\mu\text{m}$  where the target uncertainty is given by

$$U_{Target} = Tol + NORM.INV(R/2, 0, \sigma_{machine}) \quad (22)$$

where  $Tol$  is the specification limit,  $\sigma_{machine}$  is the machine tool standard deviation and the  $NORM.INV$  function has the variables; half of the target rejection rate  $R/2$ ; a mean of zero; and a standard deviation equal to the machine tool variation  $\sigma_{Machine}$ .

The dominant source of uncertainty is the worst case estimate for the seasonal thermal variation, with further consideration it is decided that tighter temperature limits can be assumed provided that the temperature in the factory is monitored. This source is therefore reduced to 11  $\mu\text{m}$  resulting in an acceptable expanded uncertainty of 26  $\mu\text{m}$ .

#### 4.2. Example 2: Hole Verification using Go/No-Go Gages

Go/No-Go plug gauges are a type of attribute gauge which gives a *pass* or *fail* result rather than a continuous measurement and therefore replicated measurements will not yield a standard deviation to be included in an uncertainty budget. It is however possible to perform attribute gage repeatability and reproducibility studies.

First the sources of uncertainty are considered and an attribute Gage R&R study is designed to include all of the relevant reproducibility conditions. 12 parts are carefully selected which have holes with diameters close to the limit for the gage

and these are calibrated on a CMM. Three operators measure each part 20 times in a random order and the results are recorded in Table 3. A least squares regression is then used to fit the normal distribution to this data and therefore calculate the mean and the standard deviation. The mean shows that there is no significant bias and the standard deviation is the total Gage R&R value to be used in the uncertainty budget.

Table 3: Results of Attribute Gage R&R Study

Calibrated Hole Diameter (mm)	Go	NOGO	% GO	Fit	Squared Difference
8.010	0	20	0%	1%	0.000
8.011	0	20	0%	3%	0.001
8.012	1	19	5%	8%	0.001
8.013	3	17	15%	17%	0.000
8.014	8	12	40%	32%	0.007
8.015	10	10	50%	50%	0.000
8.016	12	8	60%	68%	0.007
8.017	17	3	85%	83%	0.000
8.018	19	1	95%	92%	0.001
8.019	20	0	100%	97%	0.001
8.020	20	0	100%	99%	0.000
Sum of Squared Differences					0.018
Mean					8.0150 mm
Standard Deviation					0.0021 mm

The complete uncertainty budget for this process includes the total Gage R&R and the uncertainty calibration of the reference holes using a CMM. Other sources which might be considered are the drift due to the wear in the gages and thermal effects which may not have been fully represented in the Gage R&R study.

Uncertainty Source	Value	Distribution	Standard Uncertainty
Total Gage R&R	2.1 $\mu\text{m}$	Normal	2.1 $\mu\text{m}$
Reference CMM	1.7 $\mu\text{m}$	Normal	1.7 $\mu\text{m}$
Combined Standard Uncertainty			2.7 $\mu\text{m}$
Bias			0
Expanded Uncertainty (inc. bias, k=2)			5.4 $\mu\text{m}$

The expanded uncertainty should be used to set conformance limits and determine whether an acceptable level of uncertainty has been achieved in the same way as the previous example.

## 5. Conclusions

Both bottom-up uncertainty evaluations and a top-down MSA studies may miss significant sources of uncertainty. For bottom-up uncertainty evaluation significant sources of uncertainty may be missed entirely and it is the responsibility of skilled metrologists to consider all sources. With top-down MSA the validity of results depends on all influences varying in a representative way during reproducibility studies. In both cases it is necessary for all influences on the measurement result to be considered; for uncertainty evaluation so these can be explicitly included in a mathematical model; and for MSA so that study design ensures that all influences

vary representatively under reproducibility conditions. In normal industrial practice the consideration of reference standard uncertainty and thermal expansion are particular causes for concern.

The hybrid approach described in this paper provides a practical and statistically valid method of quantifying uncertainty for industrial measurements which enables conformance to be proven at a known confidence level.

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