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Optimization of Power Allocation and Sum Rate in MIMO Relay Networks

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Abstract *In this paper, we consider a MIMO relaying model for cellular networks, in which the base station communicates with far mobile users via the support of a relay. Both base station and relay are equipped with multiple antennas. Two optimization problems are investigated: maximizing the sum rate of all far users subject to the power constraints and minimizing the transmission power subject to the QoS requirement of each user. For the sum rate maximization, we propose a practical zero-forcing design scheme and optimize the power allocation to each users. For the power minimization problem, we design a zero-forcing precoders to help convert the optimization problem to a convex problem and solve it using the dual decomposition method. An iterative algorithm to obtain the optimal solution as well as the numerical experiments to support our method are also provided in this paper.*

1. Introduction

The idea of using relay to assist the communication in wireless environment is developed based on the broadcasting nature of wireless communication. The first remarkable results on capacity of relay networks was provided by Cover and Gamal from the perspective of information theory [1]. Since then, relaying has recently attracted a great interest in both industry and academic research, because it is shown to boast more desirable properties [2], [3]. Several research on optimization of the resource allocation (including time, bandwidth, power) for wireless relay networks have been conducted. For example, power allocation optimization at the PHY layer has been solved in [4]. At MAC layer, the problem of optimizing the scheduling mechanism has been investigated in [5]. A joint optimization problem of power allocation, time scheduling, and relay selection in wireless OFDM-based relay networks has also been introduced in [6]. Multiuser scheduling in a relay-assisted cellular network, in which the base station serve the near users at the same time as the relay forwards the signal to the far users is investigated by Oyman in [7]. However, all of the above research focused on single-antenna case only, because wireless networks at that time required small-size and low-power devices, which were difficult to be equipped with multiple antennas.

Nowadays, with the development of VLSI design technology, the implementation of multiple antennas is not so difficult anymore. To further improve the performance of the relay network, the MIMO technique can be integrated with the relay network. In fact, the capacity of the MIMO relay channel has been extensively investigated in [8].

One of the major problem when designing MIMO relaying model is to select an optimal precoders/beamforming vector for the source and the relay. Chae et al. [9] proposed a zero-forcing dirty paper coding (ZF-DPC) scheme to optimize the sum rate. However, the dirty paper coding involves a high complexity in implementation and is not practical. In this paper, we also investigate a zero-forcing (ZF) design scheme to optimize the sum rate of the system with lower complexity.

In another aspect, we are interested in minimizing the transmission power of the base station and the relay in the condition of keeping the minimum required QoS on each user. Particularly, in this paper we optimize the total transmission power, subject to the constraint that the signal-to-noise-plus-ratio (SINR) at each user must be maintained above certain levels. It is shown in this paper that the ZF precoder design can effectively convert this optimization problem into a convex optimization problem. Moreover, we suggest a distributed algorithm for power optimization.

The rest of the report is organized as follows. The system model and the precoder design is introduced in Section 2. Section 3 and Section 4 presents the maximization of the sum rate with the precoder design and minimizing the total transmission power, respectively. Numerical results to support the proposed method are given in Section 5. Finally, Section 6 concludes the paper.

2. System Model

2.1. General system setup

The system model of interest is illustrated in Fig. 1. The base station and its collaborating relay station are both equipped with *M* antennas. At the same time, the base station can serve *M* near users (mobile stations) and the relay can help the base station communicate with other *M* far users. So, there are at most a total of *2M* users are served simultaneously. Each user is equipped with one-antenna. We focus on the downlink transmission neglecting user selection and scheduling.

The downlink transmission is divided into two phases. In the first phase, the base station sends the information intended for the far users to the relay. In the second phase, the relay either decodes the signal or linearly processes the signal, and then forwards the processed signal to far users.

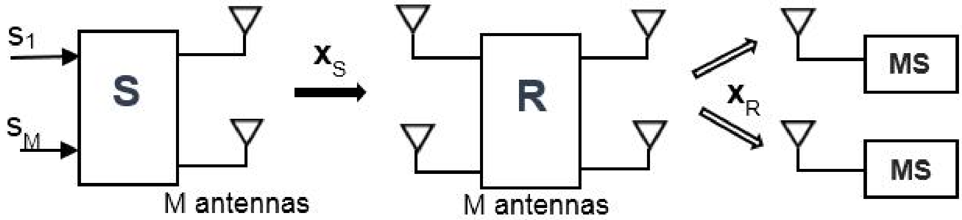

Figure 1. System model: S: Base station, R: relay, MS: mobile stations.

At relay station: the relay receives a signal following the input-output relation,

$$\mathbf{y}_d = \mathbf{H}_{sr}\mathbf{x}_s + \mathbf{z}_r, \tag{1}$$

where $\mathbf{y}_r$ is the $M \times 1$ signal vector received at the relay, $\mathbf{H}_{sr}$ is the channel matrix between the base station and the relay, $\mathbf{x}_s$ is the $M \times 1$ signal vector transmitted from the base station to the relay, and $\mathbf{z}_r$ is the complex Gaussian noise vector with distribution $CN(0, \sigma_r^2 \mathbf{I})$.

At mobile stations (far users): The received signal at the *i*th mobile user is

$$y_d^{(i)} = \mathbf{h}_{i,rd}^T \mathbf{x}_r + z_d^{(i)}, \tag{2}$$

where $\mathbf{h}_{i,rd}^T$ is the channel between the relay and the i^{th} user, $\mathbf{x}_r$ is the $M \times 1$ signal vector transmitted from the relay to the mobile users, and $\mathbf{z}_d^{(i)}$ is complex Gaussian noise with distribution $CN(0, \sigma_d^2)$. Throughout the project, we normalize the noise variance by letting $\sigma_r^2 = \sigma_d^2 = 1$.

Let $\mathbf{H}_{rd}$ be the channel matrix between the relay and the far mobile users. By using QR decomposition, we have $\mathbf{H}_{rd} = [\mathbf{h}_{1,rd} \ldots \mathbf{h}_{M,rd}]^T = \mathbf{G}_{rd}\mathbf{Q}_{rd}$, where $\mathbf{G}_{rd}$ is a lower triangular matrix and $\mathbf{Q}_{rd}$ is unitary. Similarly, let $\mathbf{H}_{sr}$ be the channel matrix between the base station and the relay, which has the singular value decomposition (SVD) given by $\mathbf{H}_{sr} = \mathbf{U}_{sr}\mathbf{\Lambda}^{\frac{1}{2}}\mathbf{V}_{sr}^H$, where $\mathbf{U}_{sr}$ and $\mathbf{V}_{sr}$ are unitary matrices, and $\mathbf{\Lambda}^{\frac{1}{2}} = \mathrm{diag}\{\sqrt{\lambda_1} \ldots \sqrt{\lambda_M}\}$.

2.2. Relaying model

For this model, we consider the relaying strategy of linear processing, in which the relay processes the received signal $\mathbf{y}_r$ with a precoder matrix $\mathbf{W}$ and forwards the signal to the destination.

Linear processing relaying: By substituting $\mathbf{x}_f = \mathbf{Fs}$ and $\mathbf{x}_r = \mathbf{W}\mathbf{y}_r$ in (1) and (2), the received signals in two phases are described as

$$\mathbf{y}_r = \mathbf{H}_{sr}\mathbf{Fs} + \mathbf{z}_r \tag{3}$$

$$y_d^{(i)} = \mathbf{h}_{i,rd}^T \mathbf{W}\mathbf{H}_{sr}\mathbf{Fs} + \mathbf{h}_{i,rd}^T \mathbf{W}\mathbf{z}_r + z_d^{(i)}. \tag{4}$$

From (4), the signal-to-interference-plus-noise ratio (SINR) for the i^{th} user is computed as

$$SINR^{(i)} = \frac{\left|\left(\mathbf{H}_{rd}\mathbf{W}\mathbf{H}_{sr}\mathbf{F}\right)_{i,i}\right|^2}{\sum_{j\neq i}\left|\left(\mathbf{H}_{rd}\mathbf{W}\mathbf{H}_{sr}\mathbf{F}\right)_{i,i}\right|^2 + \sum_{i}\left|\left(\mathbf{H}_{rd}\mathbf{W}\right)_{i,j}\right|^2 + 1}. \tag{5}$$

Hence the sum rate of the system can be calculated as

$$SR = \frac{1}{2}\sum_i \log\left(1 + SINR^{(i)}\right), \tag{6}$$

where the prelog factor is ½ because the system transmits one message in two channel uses. Our goal is to effectively design the precoders $\mathbf{F}$ and $\mathbf{W}$ such that

- the system sum rate *SR* is maximized subject to a certain power constraint on $\mathbf{F}$ and $\mathbf{W}$.
- minimize the transmission power while keeping the *SINR* for each user above a certain threshold.

Throughout this paper, we mainly utilize two precoder design schemes: zero-forcing dirty paper coding (ZF-DPC) and our zero-forcing (ZF) design. The main idea is to simplify the objective function by decomposing the matrices and convert the problem into a convex problem.

2.3. Design schemes

In this section, we review the ZF-DPC design proposed in [9] and the conventional ZF design.

ZF-DPC framework: ZF-DPC makes use of QR decomposition of the channel matrix and nulls the interference between data streams with dirty paper coding. Let

$\mathbf{W} = \mathbf{Q}_{rd}^{H}\theta^{\frac{1}{2}}$, where $\mathbf{Q}_{rd}^{H} = [\mathbf{q}_1 \ldots \mathbf{q}_M]$ and $\mathbf{\Theta}^{\frac{1}{2}} = \operatorname{diag}\{\sqrt{\theta_1} \ldots \sqrt{\theta_M}\}$. Let $\mathbf{F} = \mathbf{V}_{sr}\mathbf{P}^{\frac{1}{2}}$, where $\mathbf{P}^{\frac{1}{2}} = \operatorname{diag}\{\sqrt{p_1} \ldots \sqrt{p_M}\}$.

ZF framework: The conventional zero-forcing technique to eliminate the interference between different users is multiplying the signal vector to be transmitted with the inverse of the matrix. Let $\mathbf{F} = \mathbf{G}_d\mathbf{P}^{\frac{1}{2}}$, where $\mathbf{G}_d = \mathbf{H}_{sr}^{H}(\mathbf{H}_{sr}\mathbf{H}_{sr}^{H})^{-1}$ and $\mathbf{P}^{\frac{1}{2}} = \operatorname{diag}\{\sqrt{p_1} \ldots \sqrt{p_M}\}$. Let $\mathbf{W} = \mathbf{G}_w\mathbf{\Theta}^{\frac{1}{2}}$, where $\mathbf{G}_w = \mathbf{H}_{rd}^{H}(\mathbf{H}_{rd}\mathbf{H}_{rd}^{H})^{-1} = (\mathbf{g}_1 \ldots \mathbf{g}_M)$ and $\mathbf{\Theta}^{\frac{1}{2}} = \operatorname{diag}\{\sqrt{\theta_1} \ldots \sqrt{\theta_M}\}$.

3. Sum Rate Optimization Problem

The first optimization objective is to maximize the sum rate subject to the power constraints on the base station and the relay, which is mathematically described as

$$\begin{aligned} &\max \frac{1}{2}\sum_{i=1}^{M}\log\left(1 + SINR^{(i)}\right) \\ &\text{subject to} \quad E\left\{\left\|\mathbf{x}_d\right\|^2\right\} \le P_t;\, E\left\{\left\|\mathbf{x}_r\right\|^2\right\} \le P_r. \end{aligned} \tag{7}$$

where P_t and P_r are the maximum transmission power from the base station and from the relay. The power constraints are obtained from (7) as

$$Tr\left\{\mathbf{F}\mathbf{F}^{H}\right\} \le P_t;\, Tr\left\{\mathbf{W}\left(\mathbf{H}_{sr}\mathbf{F}\mathbf{F}^{H}\mathbf{H}_{sr}^{H} + \sigma_{sr}^{2}\mathbf{I}\right)\mathbf{W}^{H}\right\} \le P_r. \tag{8}$$

3.1. ZF-DPC design

The notations for $\mathbf{F}_d$ and $\mathbf{W}$ follow the ZF-DPC framework as defined above. By adding the received signals $y_d^{(i)}$ into a vector $\mathbf{y}_d$, we obtain

$$\mathbf{y}_d = \mathbf{G}_{rd}\boldsymbol{\theta}^{\frac{1}{2}}\mathbf{\Lambda}^{\frac{1}{2}}\mathbf{P}^{\frac{1}{2}}\mathbf{s} + \mathbf{G}_{rd}\mathbf{\Theta}^{\frac{1}{2}}\mathbf{U}_{sr}^{H}\mathbf{z}_r + \mathbf{z}_d \tag{9}$$

By using (9) to rewrite the expression of *SINR* at each mobile user *i*, where $i = 1,\ldots,M$, the sum rate of ZF-DPC design can be written as

$$SR_{ZF-DPC} = \frac{1}{2}\sum_{i=1}^{M}\log\left(1+\frac{\mathbf{G}_{rd}^2(i,i)\lambda_i\theta_i p_i}{\sum_{j=1}^{i}\mathbf{G}_{rd}^2(i,j)\theta_j+1}\right). \tag{10}$$

Here, the goal is to maximize $SR_{ZF\text{-}DPC}$ with respect to $\mathbf{p}$ and $\boldsymbol{\theta}$ subject to the following power constraints, which are derived from (8):

$$\sum_{i=1}^{M} p_i \le P_t; \quad \sum_{i=1}^{M}\theta_i(\lambda_i p_i+1) \le P_r. \tag{11}$$

Geometric programming

In high *SINR* regime, we can approximate the sum rate $SR_{ZF\text{-}DPC}$ in (10) to obtain

$$SR_{ZF-DPC} \ge \frac{1}{2}\sum_{i=1}^{M}\log_2\left(\frac{\mathbf{G}_{rd}^2(i,i)\lambda_i\theta_i p_i}{\sum_{j=1}^{i}\mathbf{G}_{rd}^2(i,j)\theta_j+1}\right) = \frac{1}{2}\log_2\left(\prod_{i=1}^{M}\frac{\mathbf{G}_{rd}^2(i,i)\lambda_i\theta_i p_i}{\sum_{j=1}^{i}\mathbf{G}_{rd}^2(i,j)\theta_j+1}\right). \tag{12}$$

Now the considered optimization problem can be equivalently rewritten as

$$\begin{aligned}&\min_{\theta,p}\prod_{i=1}^{M}\frac{\sum_{i=1}^{M}\mathbf{G}_{rd}^2(i,j)\theta_j+1}{\mathbf{G}_{rd}^2(i,i)\lambda_i\theta_i p_i}\\ &\text{subject to}\quad \sum_{i=1}^{M} p_i \le P_t; \quad \sum_{i=1}^{M}\theta_i(\lambda_i p_i+1) \le P_r.\end{aligned} \tag{13}$$

It is obvious that the objective function is a posynomial with respect to $\mathbf{p}$ and $\boldsymbol{\theta}$. As a result, the optimization is narrowed down to be geometric programming, which can be transformed to a convex problem and then solved numerically with the interior-point algorithm [10].

Equally allocated power at the base station

We obtain a lower bound of $SR_{ZF\text{-}DPC}$ by equally allocating the power at the base station, i.e., setting $p_i = P_t / M$ for $i = 1,\ldots,M$. The lower bound of $SR_{ZF\text{-}DPC}$ is derived in [9] as

$$SR_{ZF-DPC,min} = \frac{1}{2}\sum_{i=1}^{M}\log\left(1+\frac{\mathbf{G}_{rd}^{2}(i,i)\lambda_i\theta_i P_t / M}{1+\frac{P_r\left\|h_{i,rd}\right\|^2}{\lambda_i P_t / M+1}}\right). \tag{14}$$

Now the objective is to maximize $\mathrm{SR}_{ZF-DPC,\min}$ with respect to $\boldsymbol{\theta}$:

$$\max_{\boldsymbol{\theta}} SR_{ZF-DPC,min}$$
$$s.t. \quad \sum_{i=1}^{M}\theta_i(\lambda_i p_i+1)\le P_r. \tag{15}$$

The solution to this optimization problem is the well-known water-filling at the relay.

3.2. ZF Design

For the ZF framework introduced in Section 2, by adding the received signals $y_d^{(i)}$ into a vector $\mathbf{y}_d$, we obtain

$$\mathbf{y}_d = \mathbf{\Theta}^{\frac{1}{2}}\mathbf{P}^{\frac{1}{2}}\mathbf{s}+\mathbf{\Theta}^{\frac{1}{2}}\mathbf{z}_r+\mathbf{z}_d. \tag{16}$$

The *SINR* at the *i*th mobile station can be calculated as $SINR^{(i)} = \frac{\theta_i p_i}{\theta_i+1}$ for $i=1,\ldots,M$. Hence, the sum rate of ZF design is written as

$$SR_{ZF} = \frac{1}{2}\sum_{i=1}^{M}\log_2\left(1+\frac{\theta_i p_i}{1+\theta_i}\right). \tag{17}$$

By following the above frame work of the precoders and taking into account the power constraints in (8), the considered optimization problem becomes

$$(\mathbf{p},\boldsymbol{\theta}) = \arg\max_{\boldsymbol{\theta},\mathbf{p}} SR_{ZF} \tag{18}$$

$$s.t. \quad \sum_{i=1}^{M}p_i c_i \le P_t; \quad \sum_{i=1}^{M}\theta_i d_i(p_i+1)\le P_r. \tag{19}$$

where $c_i = \left(\mathbf{H}_{sr}\mathbf{H}_{sr}^{H}\right)_{i,i}^{-1}$ and $d_i = \left(\mathbf{H}_{rd}\mathbf{H}_{rd}^{H}\right)_{i,i}^{-1}$.

From here, the procedure follows the above one for ZF-DPC design. We show that the above optimization problem can be solved in two ways: geometric programming and water-filling.

Geometric programming

We obtain the approximate sum rate at high *SINR*, which is also a lower bound of the sum rate as

$$SR_{ZF} \geq \frac{1}{2}\sum_{i=1}^{M}\log_2\left(\frac{\theta_i p_i}{1+\theta_i}\right). \quad (20)$$

Similar to the derivation of (13), maximizing the approximate sum rate (20) can be converted to a geometric programming problem.

Equally allocated power at the base station

If power is equally allocated among the mobile stations i.e., $p_i = P_t / \sum_{i=1}^{M} c_i$, then a lower bound of the sum rate with ZF design is derived immediately from (20) as

$$SR_{ZF,min} = \frac{1}{2}\sum_{i=1} M\log_2\left(\frac{\theta_i P_t / \sum_{i=1}^{M} c_i}{\theta_i + 1}\right). \quad (21)$$

The objective function that maximizes the lower bound of the sum rate $\mathrm{SR}_{ZF,\min}$ with respect to $\boldsymbol{\theta}$ is given by

$$\max_{\boldsymbol{\theta}} SR_{ZF,min}$$

$$s.t. \quad \sum_{i=1}^{M}\theta_i d_i\left(1 + P_t / \sum_{i=1}^{M} c_i\right) \leq P_r.$$

Note that the solution is $\theta_i = \frac{-1+\sqrt{1+4\mu / d_i}}{2}$, where μ is to satisfy the power constraint equation (19). We can easily find μ using Newton's method.

4. Optimization of power allocation

In this section, we try to minimize the transmission power at the base station and at the relay while maintaining the QoS on each user. Here, the QoS is evaluated as the SINR requirement of each user. The optimization problem is mathematically described as

Problem 1:

$$\begin{aligned} &\text{minimize} \quad E\left\{\left\|x_f\right\|^2\right\}+E\left\{\left\|x_r\right\|^2\right\} \\ &\text{subject to} \quad SINR^{(i)} \geq \gamma_i, \forall i=1\ldots M. \end{aligned}$$

where γ_i is the threshold of the SINR level of each user *i*. Another way to formulate the problem is to minimize the transmission power of the base station subject to the power constraint on the relay which is stated as

Problem 2:

$$\begin{aligned} &\text{minimize} && E\left\{\left\|x_d\right\|^2\right\} \\ &\text{subjectto} && SINR^{(i)} \geq \gamma_i, \forall i=1\ldots M \\ &&& E\left\{\left\|x_r\right\|^2\right\} \leq P_r \\ &&& Prob\{\text{the SINR constraints cannot be satisfied}\} \leq \alpha.\% \end{aligned}$$

The probability that the SINR requirements are satisfied is closely related to the maximum relay power P_r. Hence, P_r also needs to be optimized. Our approach is to first simplify the optimization by applying the precoder design, and then convert the problem to a solvable convex optimization problem.

4.1. ZF design

From the analysis of the previous sections, the received signal for the *i^th^* mobile stations is

$$y_d^{(i)} = \sqrt{\theta_i p_i}\mathbf{s}^{(i)} + \sqrt{\theta_i} z_r^{(i)} + z_d^{(i)}. \tag{22}$$

From the analysis of the *SINR* for each user *i* in Section 3, we have $SINR^{(i)} = \dfrac{\theta_i p_i}{\theta_i + 1}$. Combining the transmission power analyzed in Section 3, we can convert the optimization Problem 1 to the following

Problem 1a:

$$\begin{aligned} &\text{minimize} && \left(\sum_{i=1}^{M}\theta_i d_i(p_i+1)+\sum_{i=1}^{M}p_i c_i\right) \\ &\text{subjectto} && \frac{\theta_i p_i}{\theta_i+1} \geq \gamma_i, \forall i=1\ldots M \\ &&& \boldsymbol{\theta} \geq 0, \mathbf{p} \geq 0. \end{aligned} \tag{23}$$

To solve this problem, we convert it to a convex optimization problem by using the fact that the objective function attains minimum only when the constraint on the SINR for each user i achieves equality. Suppose that the optimal solution does not yield an equality in the power constraint. Then we can slightly decrease p_i or θ_i such that the power constraint is still satisfied but the value for the objective function decreases. Then it is contradicted with the fact that the solution is optimal. Hence, by substituting $p_i = \gamma_i(1+\frac{1}{\theta_i})$, the problem is equivalent to

$$\max_{\theta_i \geq 0} \sum_{i=1}^{M} \left[\theta_i d_i(\gamma_i + 1) + c_i \gamma_i \frac{1}{\theta_i} \right] + \sum_{i=1}^{M} \gamma_i (c_i d_i). \tag{24}$$

We obtain the optimal solution $\theta_i = \sqrt{\frac{c_i \gamma_i}{d_i(\gamma_i+1)}}$ and $p_i = \gamma_i + \sqrt{\frac{d_i \gamma_i (\gamma_i+1)}{c_i}}$, for all $i = 1 \ldots M$.

Now consider Problem 2. First we temporarily ignore the probability constraint and will come back to find a suitable P_r meeting the probability constraint later. Hence, it is equivalent to

Problem 2a:

$$\begin{aligned} &\text{minimize} && \sum_{i=1}^{M} c_i p_i \\ &\text{subject to} && \frac{\theta_i p_i}{\theta_i + 1} \geq \gamma_i, \forall i = 1 \ldots M \\ &&& \sum_{i=1}^{M} \theta_i d_i (p_i + 1) \leq P_r. \end{aligned} \tag{25}$$

By using the same argument as in Problem 1, we conclude that in order for the objective function to attain the minimum, it must have $\frac{\theta_i p_i}{\theta_i + 1} = \gamma_i$. Hence, by substituting $p_i = \gamma_i \left(1 + \frac{1}{\theta_i} \right)$, the problem can be converted to:

Problem 2b:

$$\begin{aligned} &\text{minimize} && \sum_{i=1}^{M} c_i \gamma_i \left(1+\frac{1}{\theta_i}\right) \\ &\text{subject to} && \sum_{i=1}^{M} d_i \theta_i \left(1+\gamma_i\right) + \sum_{i=1}^{M} d_i \gamma_i \le P_r. \end{aligned} \tag{26}$$

Now this is exactly a standard convex optimization problem. One popular approach to solve this is using dual method [10]. By using the Karush-Kuhn-Tucker condition, we get the optimal dual solution $\nu = \left(\frac{\sum_{i=1}^{M} \sqrt{c_i d_i \gamma_i (\gamma_i + 1)}}{P_r - \sum i = 1M \gamma_i d_i} \right)$, and hence, the corresponding optimal primal solution is $\theta_i = \sqrt{\frac{c_i \gamma_i}{\nu d_i (\gamma_i + 1)}}$. By substituting ν in θ_i ,we have

$$\theta_i = \sqrt{\frac{c_i \gamma_i}{d - i(\gamma_i + 1)}} . \frac{P_r - \sum_{i=1}^{M} P_r - \sum_{i=1}^{M} \gamma_i d_i}{\sqrt{c_i d_i \gamma_i (\gamma_i + 1)}}, \tag{27}$$

$$p_i = \gamma_i + \sqrt{\frac{d_i \gamma_i (\gamma_i + 1)}{c_i}} . \frac{\sum_{i=1}^{M} \sqrt{c_i d_i \gamma_i (\gamma_i + 1)}}{P_r - \sum_{i=1}^{M} \gamma_i d_i}. \tag{28}$$

The next step is considering the *Prob{SINR constraints cannnot satisfy}* term.

$$\begin{aligned} &P\left\{ \sum_i \gamma_i d_i > P_r \right\} \le P\left\{ \max_i d_i > \frac{P_r}{\sum_i \gamma_i} \right\} = \\ &= 1 - P\left\{ d_i \le \frac{P_r}{\sum_i \gamma_i}, \forall i \right\} = 1 - P\left\{ \frac{2}{d_i} \ge \frac{2\sum_i \gamma_i}{P_r}, \forall i \right\} \end{aligned} \tag{29}$$

$$\overset{(a)}{=} 1 - \left[1 - F\left(\frac{2\sum_i \gamma_i}{P_r}, 2M \right) \right]^M \overset{(b)}{=} 1 - \left[1 - \frac{1}{\Gamma(M)} \gamma\left(M, \frac{2\sum_i \gamma_i}{P_r} \right) \right]^M . \quad (30)$$

where (a), (b) follows from the fact that $\frac{2}{d_i}$ are i.i.d. chi-square distributed variables with *2M* degrees of freedom and *F(x,k)* denotes the cdf function of a chi-square random variable with *k* degrees of freedom; $\gamma(s,x) = \int_0^x t^{s-1} e^{-t} dt$, and $\Gamma(M) = (M-1)!$, so we can further bound the above probability as

$$P\left\{ \sum_i \gamma_i d_i > P_r \right\} \le 1 - \left[1 - \frac{\left(\frac{\sum_i \gamma_i}{P_r} \right)^{M-1} \int_0^{\frac{\sum_i \gamma_i}{P_r}} e^{-t}}{(M-1)!} \right]^M = 1 - \left[1 - \frac{\left(\frac{\sum_i \gamma_i}{P_r} \right)^{M-1} \left(1 - e^{-\frac{\sum_i \gamma_i}{P_r}} \right)}{(M-1)!} \right]^M . \quad (31)$$

Based on this upper bound, we can find a feasible P_r, then use that P_r to solve the optimal θ_i and p_i with (27) and (28). However, this is a centralized algorithm which calls for the complete information of all the channels. In the following, we develop a distributed algorithm by making use of the dual decomposition.

Distributed iterative algorithm to Problem 2:

- Initialize the dual variable $\nu(t_0)$.
- Perform the following iteration until the solution converges:
  - With $\nu(t)$, update $\theta_i(t), i = 1 \ldots M$, as $\theta_i(t+1) = \sqrt{\frac{c_i \gamma_i}{\nu(t) d - i(\gamma_i + 1)}}$.
  - With $\theta_i(t)$, update $\nu(t)$, as $\nu(t+1) = \left[\sum_{i=1}^M \theta_i(t) d_i (\gamma_i + 1) + \sum_{i=1}^M \gamma_i d_i - P_r \right]^+$.
- After solving θ_i, we can obtain $p_i = \gamma_i \left(1 + \frac{1}{\theta_i} \right)$.

This algorithm can be regarded as an exchange of information between the relay and the users. Each user updates $\theta_i(t)$ based on the channel condition from the relay to itself d_i, the channel between the base station and the i^{th} antenna at the relay c_i, and their own threshold of *SINR* γ_i. Then the relay collects the updated θ_i from each user, computes the dual variable ν and then broadcasts it to all the users.

5. Simulation Results

In this section, Monte Carlo simulations are conducted to to evaluate the proposed algorithms. The channels between base station and relay, between relay and mobile stations (far users) are assumed to be uncorrelated Rayleigh fading channels. The near users and the far users are assumed to experience independent fading channels.

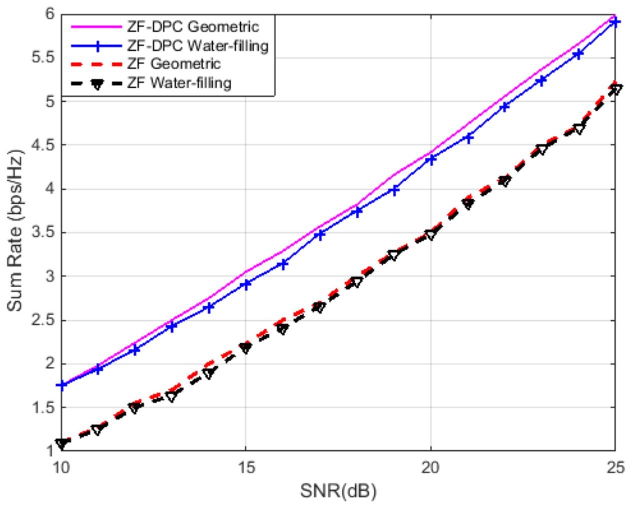

Figure 2. Maximum sum rate of ZF-DPC and ZF schemes with different solving methods and M=2

5.1. Sum rate optimization problem

In this simulation, the maximum sum rate of the systems using ZF-DPC and ZF are compared with each other. Fig. 2 shows the results when the base station and the relay are each equipped with *M = 2* antennas. As seen from the figure, ZF-DPC outperforms ZF design by approximately 0:7 bps/Hz. As the SNR increases, the gap between the sum rate of 2 schemes remains almost the same. Fig. 3 shows the corresponding results when the number of antennas is 3. Comparing Fig. 2 and Fig. 3, we can see that as the number of antenna increases, the performance gap between ZF-DPC and ZF design becomes larger.

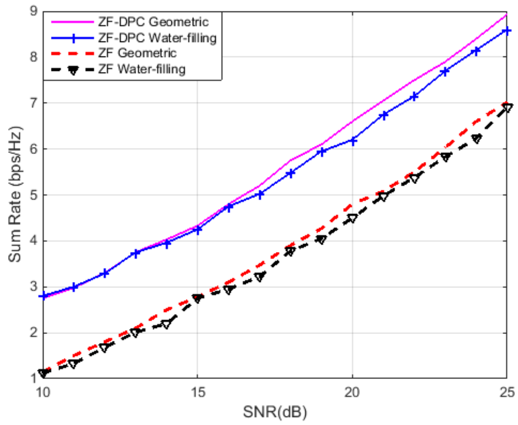

Figure 3. Maximum sum rate of ZF-DPC and ZF schemes with di erent solving methods and M=3

In both Fig. 2 and Fig. 3, there is not much difference between using geometric programming and water-filling for solving the problem. Hence, the water-filling approach might be preferred because of its lower complexity.

5.2. Power optimization problem

Fig. 4 shows the minimum total power satisfying the QoS constraint on each user as the number of users (as well as number of antennas) increases. The *SINR* threshold is set to the same value for each user. It's easily seen that the transmitting power needed will increase if either the number of users and antennas increases, or the QoS requirement for each user increases.
Fig. 5 shows the minimum power needed to satisfy two users QoS. The simulations shows the minimum power when the QoS on each user changes. It can be observed that when the *SINR* threshold on the second user γ_2 is low, the effects of the SINR threshold on the first user γ_1 is obvious. However, as γ_2 becomes very high, the difference of the minimum power needed to satisfy different γ_1 is not obvious. It is because when γ_1 is much lower than γ_2, the

minimum power needed is significantly affected by the higher *SINR* threshold, which is γ_2 in this case.

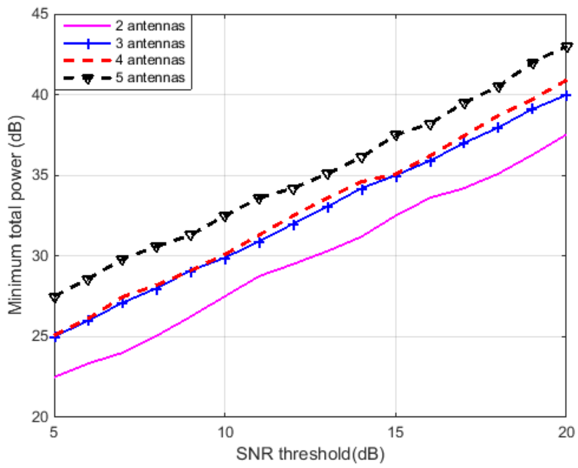

Figure 4. Minimum total transmission power

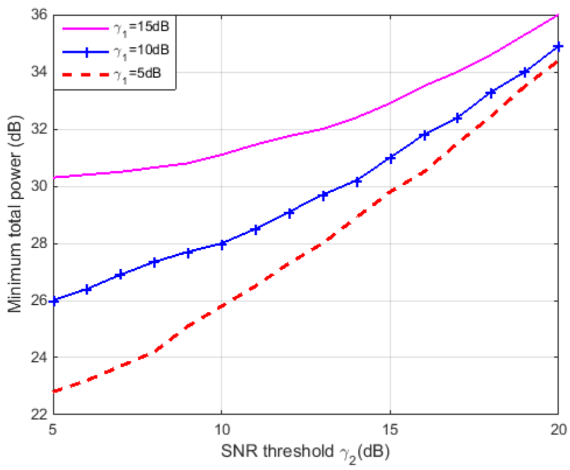

Figure 5. Minimum total transmission power with M=2

6. Conclusion

In this paper, we aim to solve two optimization problems in the relay-assisted MIMO-based cellular network. One is maximizing the sum rate subject to the power constraints; the other is minimizing the transmission power subject to the QoS constraint of each user. Two different design schemes, namely ZF-DPC and ZF, on the precoders are considered.

For the problem of sum rate maximization, the ZF-DPC design outperforms ZF design. The optimization problem can be solved by the geometric programming or water-filling method. Simulation also shows that the two optimization methods yield very close performances.

For the power minimization, two subproblems are considered: minimizing the total power at the base station and at the relay and minimizing the power at the base station subject to the power constraint on the relay power. We show that both subproblems can be converted to standard convex optimization problems. For the second subproblem, a distributed and iterative algorithm is also derived based on the dual decomposition method.

In this paper we have used classical mathematics-based approaches to find the exact solutions of the problems. While it has provided an insightful understanding of the problem, as well as provided the optimal solutions, it may take more time to execute the proposed algorithm when the number of antennas increases. This drawback can be effectively solved if we use some other meta-heuristic optimization approach as in [11]. We can confirm that there is good potential that we can combined our work with other successful mathematics-based approaches to obtain more benefit to our system model. That will be the main content of our another in-progress work.

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