Advances in Smart Systems Research ISSN 2050-8662
Vol. 4. No. 1 : pp.25-31 : isrp15-008

Birthday Hojin (Square) and Circle Hojin

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Abstract

A Hojin is an n by n square lattice with each cell containing a symbol (such as a number or a letter). Further, these numbers or letters are designed to exhibit symmetry. For example, a magic square is a Hojin where the symmetry embedded is that the numbers in each row, column and a center diagonal have an equal sum. This paper reports a new Hojin: a birthday Hojin and a circle Hojin. The birthday Hojin is the 4 by 4 Hojin with the birthday given as an eight digit numbers (four-digit number for a year; two-digit number for a month; and two-digit number for a day) will be divided four components and embedded in four cells. The circle Hojin on any refinement of a single square can include circle (ellipse) in each square (rectangle) where the circle (the ellipse) is inscribed to the square (rectangle). The sum of the circle (ellipse) area must be equal to the area of the original single circle.

1. Introduction

Hojin [1, 5] in Japanese mathematics means a magic square (or *Lo Shu* in Chinese). A Hojin is an *n* by *n* square lattice with each cell containing a number where these numbers satisfy a certain constraint: sums in each row, column, and a center diagonal are equal. More general Hojin can have other symmetries.

Several Hojins (squares) have been studied in discrete mathematics [3, 8-11], typified by Latin squares and Greco-Latin squares (Euler squares) [2]. As several names such as Hojin, Lo Shu, and Squares indicated, they have been extensively studied in world wide ([1] in Japan for example) and for a period of historic scale. Greco-Latin squares have been used in the experimental design to make sure all the possible combination of the control factors are involved. As for Latin square, 9x9 two layered latin square are used a puzzle known as Sudoku [4].

On the other hand, cellular automata (CA) have been attracting attention as a potential model for complex systems (e.g., [12]). CA also can be expressed as a square lattice where each cell can take one state among several states. One difference between CA and Hojin is that the former is a dynamical system, while the latter is a static one.

In order to bridge between CA and Hojin, this paper tries a preliminary study to design a dynamic Hojin that changes by a certain rule while satisfying the symmetry of Hojin. Since there is much degree of freedom in a design of the dynamic Hojin, this paper can present an example of the dynamic Hojin based on symmetric Hojins.

3	73	213	223	6	185	256	76	8	248	178	78	2	212	52	242
232	203	63	13	66	246	196	16	228	48	38	198	232	62	182	32
243	193	53	23	236	36	46	206	68	188	238	18	202	12	252	42
33	43	183	253	216	56	26	226	208	28	58	218	72	222	22	192
197	57	237	17	99	158	103	154	131	114	143	126	30	10	250	230
187	27	227	67	90	155	102	167	134	123	130	127	240	220	20	40
77	217	37	177	168	101	156	89	124	133	128	129	190	210	70	50
47	207	7	247	157	100	153	104	125	144	113	132	60	80	180	200
88	176	160	92	136	105	121	152	135	122	106	151	87	159	175	91
164	84	96	172	146	119	107	142	118	147	139	110	97	173	161	81
98	162	174	82	117	140	148	109	145	108	120	141	163	95	83	171
166	94	86	170	115	150	138	111	116	137	149	112	165	85	93	169
15	25	225	255	21	1	231	251	29	9	249	229	14	44	214	244
235	245	5	35	221	241	11	31	239	219	19	39	204	254	4	54
65	55	215	185	211	191	61	41	189	209	69	49	224	194	64	34
205	195	75	45	51	71	201	181	59	79	179	199	74	24	234	184

Figure 1. A Hojin Composed by Gakuho Abe

Section 2 presents Hojin with specific numbers embedded, giving examples of Gakuho Hojin and the 4x4 birthday Hojin. Section 3 briefly states the studies of Hojin with geometric figures placed instead of numbers. Among geometric Hojins, we focus on circle Hojins.

2. Embedding Specific Numbers In Hojin

2.1. Gakuho Hojin

Gakuho Abe is a composer of Hojin [1]. Among many types of Hojins he composed, an interesting one is a multi-scale Hojin. As shown in Figure 1, entire Hojin consists of 16 of 4 by 4 Hojins where the constant sum of each Hojin, in turn, forms 4 by 4 higher level Hojin.

2.2. Birthday Hojin

Specific numbers may be embedded in Hojin. For example, birthday when expressed as some digit numbers can be embedded. Figure 2 shows two examples where the date 113.4.16 is embedded with the constant sum 34 in the diagonal (left) and the date 1605.2.11 with the constant sum 34 in the top row (right). These two Hojins come from a collection of dynamic Hojin [11].

A challenge for this type of birthday Hojin is how to handle the case when same numbers appeared as in 2001.1.1, for Hojin requires distinct numbers for each cell. There are many ways to embed numbers in Hojin.

1	12	15	6
8	13	10	3
14	7	4	9
11	2	5	16

16	5	2	11
9	4	7	14
3	10	13	8
6	15	12	1

Figure 2. Birthday HOJINs, e.g., for the birthdays:
0113.04.16 diagonal (left) and
1605.02.11 top row (right)

3. Geometric Hojin

Geometric Hojin is a new type of Hojin that can include geometric figure in each cell instead of numbers. Because a geometric figure is included each cell, the Hojin can have several regularities other than the constraint satisfied by the magic square: the equal sum in row, column, and center diagonal. We can build an example of 4x4 dynamic Hojin based on 4x4 symmetric Hojin. The regularity can

be geometric symmetries as in symmetric Hojin (section 3.1). Other regularities include that the constraint of the equal sum of the area of the circles in one Hojin as in circle Hojin (section 3.2).

3.1. Symmetric Hojin

In Hojin, each cell usually holds a number, however, it can be a symbol or any other objects as far as they can have some specific relation [5]. Indeed, Latin squares and Greco-Latin Squares hold letters. The study of symmetric Hojin suggested the Hojins themselves would exhibit a geometric symmetry (when letters are considered as geometric pattern as in Figure 3 below and left), although Hojin can exhibit position symmetry (when letters are considered as literally letters as in Figure 3 above and right).

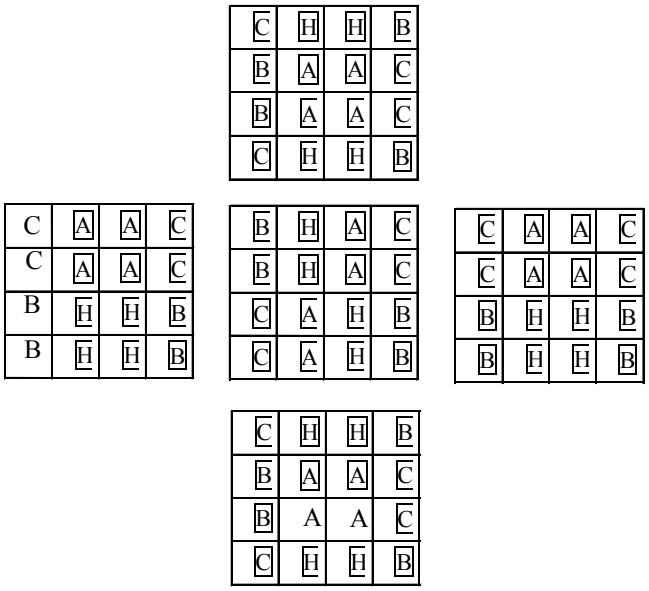

Figure 3. Symbol HOJINs arranged as a Mandara form.

3.2. Circle Hojin

Geometric Hojin holds a geometric object in each cell instead of numbers. Another example of the geometric Hojin is a circle (and ellipse) Hojin [6] whose cells hold a geometric object of circle.

Circle Hojins can be composed by two primitive processes: a refinement with squares where a circle is inscribed inside (Figure 4); and a refinement with rectangles where an ellipse is inscribed inside (Figure 4 *below*). Composite circle

Hojins may be built by combination of these two processes with distinct scales (Figure 4 *left* and *right*).

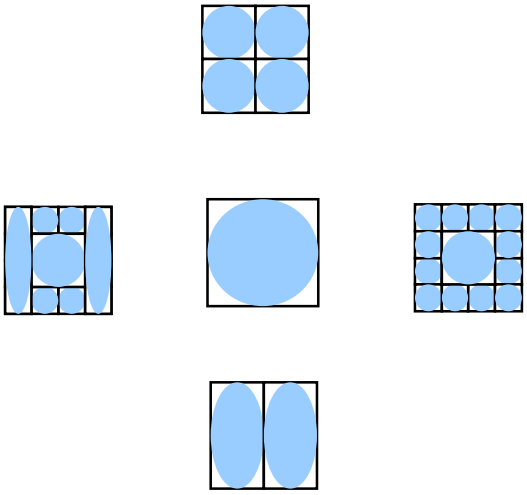

Figure 4. Circle HOJINs arranged as a Mandara form.
The sum of circle area of these Hojins are identical.

By sequentially applying these processes, many beautiful patterns can be drawn. For example, Figure 5 left and right can be drawn by applying the original circle Hojin in Figure 5 center. Again, the sum of circle area are equal for these three circle Hojins in Figure 5.

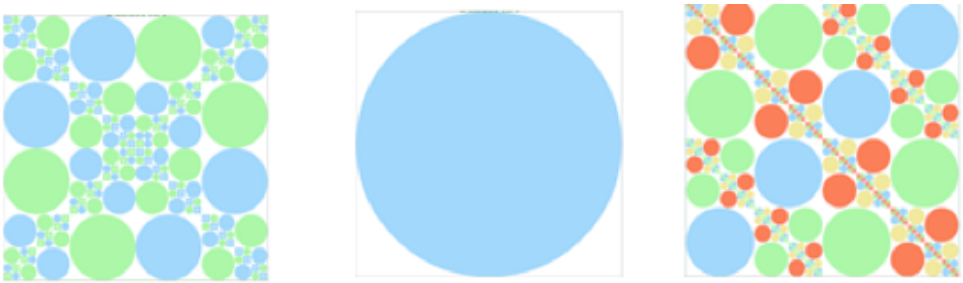

Figure 5. Two examples of beautiful circle HOJINs (*left* and *right*)
where both are obtained by applying refinement
to the original circle Hojin (*center*).

4. Discussions

The way of building the Birthday Hojin proposed a problem of composing Hojin when given any distinct four numbers. The challenge of Birthday Hojin: the four

numbers may not be distinct when the date expressed as four consecutive numbers inspired us several ways of embedding numbers in Hojin.

The Circle Hojin, on the other hand, inspired us to design a design tool that allows users to solve the problem of how to draw any given painting by combining and arranging primitive operations of refinement.

As for the significance of the work for intelligent systems, we need to remind ourselves that design problems are of vital importance due to the difficulty in attaining creativity by machines. The mathematical objects such as Hojins may be considered as an intelligent media that allows human to inspire not only academic objects but also artistic objects.

5. Conclusions

An attempt to develop new types of Hojin (square) has been made with an emphasis that Hojins would have external symmetry in the relation among Hojins other than the internal symmetry so far studied.

Further, since Hojin is a mathematical object similar to numbers, each Hojin could have a specific character similarly to a specific numbers. Composing new type of Hojin is a design mathematics that is required for education in current Engineering and Science.

Acknowledgments

We would like to thank the reviewers for their valuable comments. Discussion on the importance and significance of the work in relation to intelligent systems has been added, reminded by their comments.

The Hojins presented here have been often exhibited at the “WASAN Museum” in which many other mathematical objects visualized by information and communication technology (ICT) have been exhibited. The WASAN Museum also has been organized to support several annual projects such as the Super Science High Schools (SSH) by Japan Science and Technology Agency (JST); and the Junior Science Course held at the Toyohashi Museum of Natural Resources by the local board of education.

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