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Consensus based scheduling of storage capacities in a virtual microgrid

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Abstract

We present a distributed, decentralized method for coordinated scheduling of charge/discharge intervals of storage capacities in a utility grid integrated microgrid. The decentralized algorithm is based on a consensus scheme and solves an optimisation problem with the objective of minimising, by use of storage capacities, the power flow over a transformer substation from/to the utility grid integrated microgrid. It is shown that when using this coordinated scheduling algorithm, load profile flattening (peak-shaving) for the utility grid is achieved. Additionally, mutual charge/discharge between batteries which are interconnected in the same grid is prevented. The effect of coordinated scheduling and the resulting prevention of mutual charge/discharge is validated by a benchmark simulation.

1 Introduction

It is envisioned that the future energy eco-system will extend the existing electrical grid with additional functionalities on top of the existing energy supply layer. This layer-extension can provide functionalities such as smart metering, real-time monitoring, grid balancing and local energy management. It can, as such, act as a middleware layer between the existing grid structure components and the overlying Smart-Grid applications for wholesale and retail market mechanisms [2]. This structure can be used by distribution grid operators (DSO) for operation of a 'Smart-Grid'. In this paper, we introduce a decentralized distributed scheduling method which can be implemented as part of middleware layer implemented by a MAS, to schedule charging/discharging of distributed storage devices in microgrids. Virtual microgrids are envisioned as being part of the existing distribution grid infrastructure (downstream of distribution substations). They can be implemented on top of the existing infrastructure of a supply grid, preferably in low and medium voltage grids to act as an intermediate layer between existing grid infrastructure and overlying applications. A virtual microgrid (VMG) bundles multiple local, installed on the same grid layer, distributed energy generation units and local loads as well as storage capacities, to work as a virtual sub-grid in parallel to the utility grid. The main objective of a VMG is to operate a cluster of prosumers (a load with co-located generation unit) and storage capacities within a topological layer of the utility grid to increase the level of self-consumption.

In order to ensure a secure supply within a VMG, all integrated active and dispatchable supply system components have to be operated in a coordinated way and a common dispatch plan has to be compiled on the basis of predictions for load profiles, production predictions and electricity prices. This is achieved by using an energy management system (EMS) which is part of a control system for a VMG. The EMS is responsible for optimising the operation of the local VMG towards given operational objectives determined by the overlying layer applications. In a decentralised architecture such as a Multi-Agent System (MAS) each node is aware of local information on technical performance characteristics, constraints and requisites which it then contributes to a cooperative decision making process. Thus making a MAS an ideal candidate for implementation of a distributed energy management system. Additionally, a MAS ensures modularity and compatibility through standardised interfaces to act as a distributed middleware layer between the existing grid infrastructure and overlying application layers. It is shown in the following, how MAS based coordinated scheduling of storage capacities of a VMG inside a common low voltage (LV) grid, in contrast to battery scheduling with respect to a local perspective only, improves utilisation of battery resources since batteries are operated synchronously and mutual charging/discharging is prevented.
The paper then continues with an overview of related work, providing an identification of the problem of mutual charging/discharging of batteries in a VMG. It is shown that the problem can be formulated as a mathematical optimisation problem and which methods are used to solve it, and how mutual charging/discharging can be prevented by inter-agent coordination based on the consensus algorithm. The final part of the paper illustrates the concept of the introduced method by a benchmark simulation.

2 Related Work

A review of MAS based modelling and simulation of the future electricity grids and markets and an introduction to the layered structure of a future grid system is given by [13, 2]. A comprehensive overview of MAS for application in power systems engineering is given in [8, 9]. The problem of finding an operational schedule for charge and discharge periods for a local battery co-located to a local energy generation unit can be formulated as an optimisation problem which assumes that forecasts for the scheduling horizon are given. Approaches have been introduced based on the concept of dynamic tariffs where the objective is economical cost reduction [4] based on a price change for energy bought from the grid. The problem formulation can be focused on the perspective of residential customers [12, 5, 14] but also on the system operators perspective [6]. Other approaches use as an objective, the reduction of peaks in the demand profile with the goal of minimising the energy flow from/to the utility grid to the consumer [10]. It can be shown in the following, that problems occur in the case where several EMS for battery scheduling, which do not cooperate, are implemented in the same grid e.g. a residential grid. Mutual charge and discharge of batteries occurs if EMS on the same grid do not coordinate their charge and discharge scheduling.

3 Coordinated Scheduling of Distributed Storage Capacities

As the objective of this distributed EMS scenario, we have chosen the minimisation of power flow over a substation to an overlying grid such as e.g. a medium-voltage (MV) grid. To achieve this aim, a system of decentralised storage capacities is managed by a decentralised EMS based on a low level MAS. Each of the EMS agents is able to control power flow vectors to either charge or discharge local storage capacities. Also, each supply component which is managed by an agent is connected to a common LV grid. Such a scheme is provided, for example, in LV residential grids. As previously suggested by [12], the chosen objective in the coordinated optimisation problem is the minimisation of energy flow from/to the overlying grid using as objective, the minimisation of a quadratic objective function:

$$\sum_{i=1}^{m}\sum_{j=1}^{s}\left(\psi_i^j\right)^2 \tag{1}$$

Here m denotes the number of the distributed EMS nodes i, $\forall i \in \{1,\ldots,m\}$, interconnected in a distributed prosumer system, s is the number of equal time intervals of length Δt the scheduling time horizon W is divided by. Further ψ_i^j, $\forall j \in \{1,\ldots,s\}$, denotes the averaged power in kW supplied from, or fed to the grid by node i over a period $[t_{j-1}, t_j]$. Thus, the power flow profile vector from/to the grid is denoted by $\boldsymbol{\psi}_\mathbf{i} \in \mathbb{R}^s$ with $\boldsymbol{\psi}_i = [\psi_i^1,\ldots,\psi_i^s]^T$. The flow of power from and to peripheral components controlled by a local EMS agent such load, production and storage in kW is represented by flow vectors, all of length s. The following power flows are defined:

Load power flow profile, $\mathbf{l}$: The averaged power flow in kW at node i to local loads over a period $[t_{j-1}, t_j]$ is denoted by l_i^j, $\forall j \in \{1,\ldots,s\}$ and $\forall i \in \{1,\ldots,m\}$. Thus the vector of power flow to local loads at node i is denoted by $\mathbf{l}_i \in \mathbb{R}^s_{\geq 0}$, with $\mathbf{l}_i = [l_i^1,\ldots,l_i^s]^T$.

Production power flow profile, $\mathbf{p}$: The averaged power flow in kW at node i from local generation units over the period $[t_{j-1}, t_j]$ is denoted by p_i^j, $\forall j \in \{1,\ldots,s\}$ and $\forall i \in \{1,\ldots,m\}$. Thus the produced power profile vector of local generation units at node i is denoted by $\mathbf{p}_i \in \mathbb{R}^s_{\geq 0}$, with $\mathbf{p}_i = [p_i^1,\ldots,p_i^s]^T$.

Storage power flow profile, $\mathbf{b}$: The local estimate of the averaged power flow in kW at node i into and out of its storage capacities over the period $[t_{j-1}, t_j]$ is denoted by b_i^j, $\forall j \in \{1,\ldots,s\}$ and $\forall i \in \{1,\ldots,m\}$. Thus the local estimate of the power flow profile vector at node i into and out of it's storage capacities in kW is denoted by $\mathbf{b_i} \in \mathbb{R}^s$, $\mathbf{b}_i = [b_i^1,\ldots,b_i^s]^T$. Note that if $b_i^j \in \mathbb{R}_{>0}$ or if $b_i^j \in \mathbb{R}_{<0}$ the battery is charging or discharging, respectively.

Having defined the local power flow vectors, a local energy balance for each node can be defined as:

$$\hat{\mathbf{b}}_i + \boldsymbol{\psi}_i = \mathbf{l}_i - \mathbf{p}_i, \qquad \forall i \in \{1, \dots, m\}. \tag{2}$$

The overlying interaction topology of EMS agents in a communication network is represented by the graph $G = (\mathcal{V}, \mathcal{E})$ with the set of EMS agents $\mathcal{V} = \{1, \dots, m\}$ and a set of edges $\mathcal{E}$ denoted by (i, j), with $(i, j) \in \mathcal{V}$. In the given EMS system, communication between two agents will occur at discrete times instances. The method is based on the consensus average algorithm [11], where each EMS agent exchanges with it set of neighbour agents $\mathcal{N}_i$ its current estimate on the storage power flow profile vector $\hat{\mathbf{b}}_i$ relative to capacity limit of the individual battery. The relative storage power flow profile vector for each node i can therefore be defined, $\forall i \in \{1, \dots, m\}$ as:

$$\hat{\mathbf{r}}_i = \frac{\hat{\mathbf{b}}_i}{C_i}. \tag{3}$$

Based on the consensus algorithm, each EMS agent i iteratively exchanges its local estimate of $\hat{\mathbf{r}}_i$ with its neighboring agent such that:

$$\hat{\mathbf{r}}_i^{(k+1)} = \sum_{j=1}^{m} w_{ij} \hat{\mathbf{r}}_j^{(k+1)} \tag{4}$$

where w_{ij} denotes the (ij)-element of a row stochastic weight matrix $\mathbf{W}$ with $w_{ii} > 0$ and $0 \leq w_{ij} \leq 1, \forall j \neq i$. The upper and lower limits for the estimate of the power flow into and out of the storage capacity are treated by a maximum and minimum charging, respectively discharging constraint which is defined by the individual storage capacity in use, thus:

$$B_i^- \leq \hat{b}_i^j \leq B_i^+, \qquad \forall j \in \{1, \dots, s\}, \forall i \in \{1, \dots, m\}. \tag{5}$$

Here $B_i^- \in \mathbb{R}_{\leq 0}$ denotes the maximum discharge power and $B_i^+ \in \mathbb{R}_{\geq 0}$ the maximum charge power of each individual storage capacity at node i. The state of charge of a battery in kWh at node i is represented by $\boldsymbol{\xi}_i = (\xi_i^0, \dots, \xi_i^s)$ with $\boldsymbol{\xi}_i \in \mathbb{R}_{\geq 0}^{s+1}$. Where ξ_i^0 denotes the initial and ξ_i^s the final state of charge, respectively. The state of charge at a time $t = s\Delta t$ is:

$$\xi_i^s = \xi_i^0 + \sum_{j=1}^{s} \hat{b}_i^j \Delta t, \qquad \forall i \in \{1, \dots, m\}, \tag{6}$$

which is constrained by the capacity C_i of the storage capacity, such that:

$$0 \leq \boldsymbol{\xi}_\mathbf{i} \leq C_i \begin{bmatrix} 1 \\ \mathbb{1} \end{bmatrix}, \qquad \forall i \in \{1, \dots, m\}. \tag{7}$$

Here $\mathbb{1} \in \mathbb{R}^s$ denotes an all-1 column vector of length s. For simplicity over the scheduling time horizon W it is assumed that the final state of charge should equal the initial state of charge such that:

$$\xi_i^s = \xi_i^0, \text{with: } 0 \leq \xi_i^0 \leq C_i, \qquad \forall i \in (1, \dots, m), \tag{8}$$

and:

$$\sum_{j=1}^{s} \hat{b}_i^j = 0, \qquad \forall i \in 1, \dots, m. \tag{9}$$

In order to formulate an optimisation problem subject to the above introduced constraints, the following vector of decision variables is defined for each EMS node i:

$$\mathbf{x_i} = \begin{bmatrix} \boldsymbol{\psi}_\mathbf{i} \\ \mathbf{b_i} \end{bmatrix}, \qquad \forall i \in \{1, \dots, m\}. \tag{10}$$

On the basis of the vector of decision variables the introduced constraints in (5), (7) and (8) can be combined into i local linear inequality constraints such that:

$$\mathbf{B}_i \mathbf{x}_i \leq \boldsymbol{\beta}_i, \qquad \forall i \in \{1, \dots, m\}, \tag{11}$$

with $\mathbf{x}_i \in \mathbb{R}^{2s}$, and:

$$\mathbf{B}_i = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{0} & -\mathbf{I} \\ \mathbf{0} & \mathbf{T} \\ \mathbf{0} & -\mathbf{T} \end{bmatrix}, \quad \boldsymbol{\beta}_i = \begin{bmatrix} B_i^c \\ B_i^d c \\ \left(\frac{\mathbb{1}}{\Delta t}\right)(C_i - \xi_i^0) \\ \left(\frac{\mathbb{1}}{\Delta t}\right)\xi_i^0 \end{bmatrix}, \tag{12a}$$

where $\mathbf{B}_i \in \mathbb{R}^{4s \times 2s}$ and $\boldsymbol{\beta}_i \in \mathbb{R}^{4s}$, $\forall i \in \{1, \dots, m\}$. Here $\mathbf{I} \in \mathbb{R}^{s \times s}_{\geq 0}$ denotes an identity matrix, $\mathbf{0} \in \mathbb{R}^{s \times s}$ a null matrix and $\mathbf{T} \in \mathbb{R}^{s \times s}_{\geq 0}$ a lower binary triangular matrix of all-1s with the elements such that $t_{uv} = 1$ for all $u \geq v$, and 0 otherwise. Here u and v denote the row and column indices. Similarly (2) and (9) can be combined into local linear equality constraint such that, $\forall i \in \{1, \dots, m\}$:

$$\mathbf{A}_i \mathbf{x}_i = \boldsymbol{\alpha}_i, \tag{13}$$

with:

$$\mathbf{A}_i = \begin{bmatrix} \mathbf{0}^T & \mathbb{1}^T \\ \mathbf{I} & \mathbf{I} \end{bmatrix}, \qquad \boldsymbol{\alpha}_i = \begin{bmatrix} \mathbf{0} \\ \mathbf{l_i} - \mathbf{p_i} \end{bmatrix}, \tag{14}$$

with $\mathbf{A}_i \in \mathbb{R}^{(s+2) \times 2s}$ and $\boldsymbol{\alpha}_i \in \mathbb{R}^{(s+2)}$ and $\mathbf{0}$ an all-0 column vector. Based on the objective function (1), the inequality constraints (12) and the equality constraints (14), the following local quadratic optimisation problem can be defined for each EMS node i, $\forall i \in \{1, \dots, m\}$:

$$\min_{\mathbf{x_i}} \frac{1}{2} \mathbf{x_i}^T \mathbf{Q_i} \mathbf{x_i} \tag{15a}$$

subject to:

$$\mathbf{A}_i \mathbf{x_i} = \boldsymbol{\alpha}_i, \tag{15b}$$

$$\mathbf{B}_i \mathbf{x_i} \leq \boldsymbol{\beta}_i. \tag{15c}$$

Where $\mathbf{Q}_i \in \mathbb{R}_{\geq 0}^{2s \times 2s}$ denotes the following quadratic terms coefficients matrix:

$$\mathbf{Q}_i = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \forall i \in \{1, \ldots, m\}. \tag{16}$$

With $\mathbf{I}$ being an $s \times s$ identity matrix.

3.1 Decentralised coordinated scheduling algorithm

The algorithm for solving the optimisation problem as shown in Problem (15) is based on Lagrangian duality where the objective function (15a) is augmented with a weighted sum of the constraints functions (15b) and (15c). Thus, the associated Lagrangian L_i to the individual optimisation Problem 15 for each node i becomes, $\forall i \in \{1, \ldots, m\}$:

$$L_i(\mathbf{x_i}, \boldsymbol{\lambda}_i, \boldsymbol{\mu}_i) = \frac{1}{2}\mathbf{x}_i^T \mathbf{Q_i} \mathbf{x}_i + \boldsymbol{\lambda}_i^T(\mathbf{A}_i \mathbf{x}_i - \boldsymbol{\alpha}_i) + \boldsymbol{\mu}_i^T(\mathbf{B}_i \mathbf{x}_i - \boldsymbol{\beta}_i), \tag{17}$$

where $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$ are the Lagrange multipliers. Based on Slaters Theorem, for a strictly feasible and convex primal problem sufficient conditions for strong duality hold and the solution to the Lagrange dual is a lower bound to the optimal value of the original problem. The equilibrium saddle point Karush-Kuhn-Tucker (KKT) conditions of the Lagrangian can be described as the following saddle point system:

$$\underbrace{\begin{bmatrix} \mathbf{Q}_i & \mathbf{A}_i^T & \mathbf{B}_i^T \\ \mathbf{A}_i & 0 & 0 \\ \mathbf{B}_i & 0 & 0 \end{bmatrix}}_{\text{M}} \begin{bmatrix} \mathbf{x_i^*} \\ \boldsymbol{\lambda}_\mathbf{i}^* \\ \boldsymbol{\mu}_\mathbf{i}^* \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\alpha}_\mathbf{i} \\ \boldsymbol{\beta}_\mathbf{i} \end{bmatrix}, \tag{18}$$

with the saddle point matrix $\mathbf{M}$, $\forall i \in \{1, \ldots, m\}$. To converge to a saddle point, the primal and dual gradient method known as Arrow-Hurwicz-Uzawa (AHU) algorithm [1] is used which searches for the saddle-point of the Lagrangian by alternating steps in the direction of the gradients:

$$\mathbf{x}_i^{(k+1)} = \mathbf{x}_i^k - \tau \left[\mathbf{Q}_i \mathbf{x}_i^{(k)} + \mathbf{A}_i^T \boldsymbol{\lambda}_i^{(k)} + \mathbf{B}_i^T \boldsymbol{\mu}_i^{(k)}\right], \tag{19a}$$

$$\boldsymbol{\lambda}_i^{(k+1)} = \boldsymbol{\lambda}_i^k + \upsilon \left[\mathbf{A}_i \mathbf{x}_i^{(k+1)} - \boldsymbol{\alpha}_i\right], \tag{19b}$$

$$\boldsymbol{\mu}_i^{(k+1)} = \boldsymbol{\mu}_i^k + \upsilon \left[\mathbf{B}_i \mathbf{x}_i^{(k+1)} - \boldsymbol{\beta}_i\right]^+. \tag{19c}$$

where the scalar $\tau > 0$ and $\upsilon > 0$ are the primal and dual constant stepsizes and $[]^+$ denotes a projection on the positive orthant $\mathbb{R}_{\geq 0}^s$, such that $\mu_i^j = max\{\mu_i^j, 0\}, \forall j \in \{1, \ldots, s\}$. The AHU-algorithm is known to be convergent under special conditions (convexity-concavity of the Lagragian) and for special step sizes τ and υ [7] which can be determined based on convergence analysis of the underlying basic gradient algorithm [3, 15].

4 Benchmark simulation

To illustrate the concept of coordinated scheduling, we consider a scenario of three different residential households. Each is equipped with local energy generation units based on PV installations, as well as co-located battery storage capacities with different capacities. Figure 1 shows the modeled consumption and production profiles as simulated for the three households. Each household is equipped with one agent which solves, by gradient ascent/descent method, the local optimization problem as described in (15). It is assumed that these households are equipped with PV panel installations and local battery capacities with limited charge and discharge currents and limited storage capacity and that they are connected on the same LV grid. Each agent is able to control, in each household: the energy flow in and out of the battery and iteratively exchange that with neighboring agents as described in (4) and shown in Figure 1 (right-hand side). All agents will thus converge to a consensus for each time interval, so that either all batteries are charged or discharged and mutual charging/discharging is prevented.

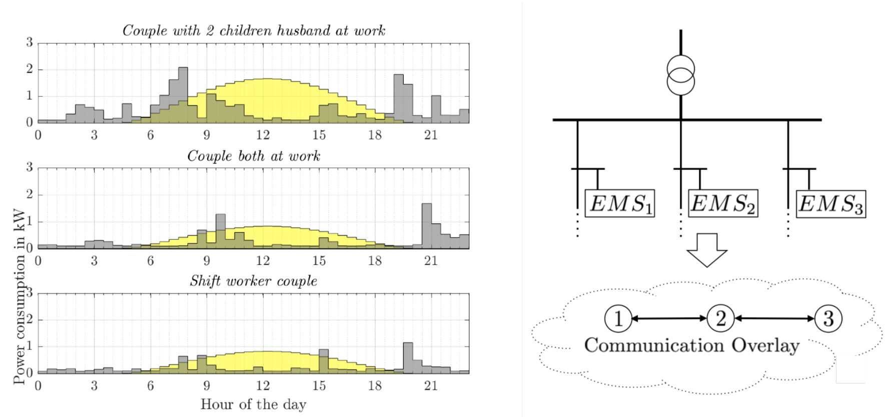

Figure 1: Simulated load (grey) and local PV production profiles (yellow) for three different households (left-hand side) connected on the same LV grid (right-hand side). These are operated by three individual agents which iteratively exchange local estimates of $\hat{\mathbf{r}}_i$ over a communication overlay network (right-hand side).

Results The results obtained for the given optimisation Problem 15 are presented in Figure 2. This shows the optimal battery power flow $b_i^j, \forall j \in \{1, \ldots, s\}$ over the scheduling time horizon window W as a stacked bar graph for each battery (in different grey-scales) as cooperatively agreed among the EMS agents using the coordinated scheduling algorithm. Further shown in Figure 2 is the original grid load profile (sum over all loads) and the residual grid profile after coordinated and non-coordinated scheduling, respectively. The results as shown in the left chart of Figure 2 can be compared with the right chart of Figure 2 which shows the data for the same same load and production profile simulation. However, in contrast to the coordinated MAS scheduling, here each agent solves the given optimisation problem without cooperation among agents. The results obtained in these simulations

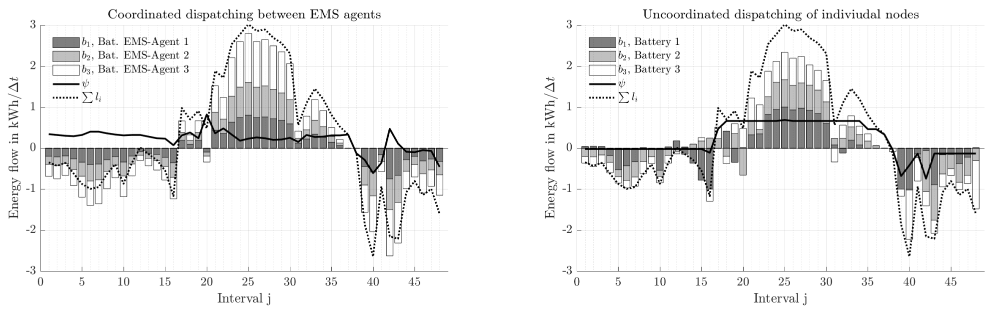

Figure 2: Coordinated battery charge/discharge profile (left-hand side) $\mathbf{b_i}$ as agreed upon EMS agents with grid load profiles and uncoordinated battery charge/discharge profile (right-hand side) $\mathbf{b_i}$ for individual EMS nodes without cooperation among each other. Simulated load profile $\mathbf{l}_i$ (dashed line) and residual grid load profile ψ_i (solid line) after dispatching.

demonstrate a significant reduction in magnitude of the grid load profile, thus the feed-in peaks around noon as well as the peak loads in the morning and evening are smoothed. This is evident for both simulation scenarios. However, there are two significant differences between the two scenarios. (1) The coordinated scheduling operates the distributed storage capacities as one virtual storage capacity, which results in a more flattened residual grid load profile in contrast to the case of the uncoordinated scheduling. Thus, the residual grid load is more evenly distributed over the scheduling time horizon. (2) In contrast to the results obtained with a coordinated scheduling algorithm, when optimising individually only, charge and discharge intervals between the different nodes are not coordinated. Cases are present where in the same time interval j, a battery at a particular node charges whereas a battery at a different node discharges, so mutual charging/discharging occurs. This can be seen in the range of $j = 14$ to 20 and others in Figure 2 (left-hand side).

5 Conclusions

A decentralised energy management system for coordinated scheduling of storage capacities inside a virtual microgrid was introduced. It is indicated that mutual charging/discharging of batteries occurs if these are operated by EMS agents which compile charge/discharge schedules based on local information only. A method, based on the consensus algorithm and inter-agent communication is introduced to prevent mutual charging/discharging. Using this method, all agents converge to a common charge/discharge schedule.

Based on a benchmark simulation, it is illustrated that through the use of the consensus algorithm, mutual charge/discharge of batteries is prevented. This highlights the advantage of coordination in a distributed EMS for scheduling of distributed storage capacities and underline the differences to the uncoordinated case. Further, it is shown that coordinated MAS based scheduling on the basis of residual grid load profile optimisation can significantly improve grid load by providing peak-shaving over a common transformer substation. In progress is currently an extended scheme in which an overlying supervisory authority agent can dictate a set-point for the utilization of the batteries inside the VMG so that the amount of peak shaving can be influenced. Additionally a predefined profile can be dictated by a supervisory agent and so that the EMS agents cooperatively optimise towards that profile. This then reflects a least-squares function approximation, with the objective of minimising the difference between the given profile (function) and the profile provided by the VMG.

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